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**Citation for published version:**

Burguet, R & Sakovics, J 2017, 'Competitive foreclosure', *The RAND Journal of Economics*, vol. 48, no. 4, pp. 906-926. <https://doi.org/10.1111/1756-2171.12206>

**Digital Object Identifier (DOI):**

[10.1111/1756-2171.12206](https://doi.org/10.1111/1756-2171.12206)

**Link:**

[Link to publication record in Edinburgh Research Explorer](#)

**Document Version:**

Peer reviewed version

**Published In:**

The RAND Journal of Economics

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# Competitive foreclosure\*

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February 15, 2017

## Abstract

We present a model where oligopolistic firms producing substitutes compete for inputs in a decentralized market. Input suppliers are capacity constrained (or produce under exclusivity). Compared to a price-taking input market, the incentive to foreclose downstream competitors not only leads to higher input prices, but it also results in a higher aggregate amount of input acquired. This novel feature mitigates the output reducing effect of downstream market power and may even restore efficiency in the unique (input) market clearing equilibrium. Other equilibria where firms endogenously coordinate on which suppliers to target result in excess input supply (involuntary unemployment, if input is labor) and even higher input prices. Our insights generalize to alternative vertical structures. *JEL* Codes: D43, L11, L13.

Key words: simultaneous auctions, targeted offers, vertical linkages, involuntary unemployment.

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\*This paper supercedes "Bidding for input in oligopoly". We thank Simon Board, Ramon Caminal, Michael Elsby, Juanjo Ganuza, Philipp Kircher, Laurent Lamy, Carmen Matutes, John Moore, Katharine Rockett, Ilya Segal, Michelle Sovinsky and Ludo Visschers – as well as audiences at EARIE 2016, Jornadas de Economía Industrial 2016, Barcelona GSE, Paris SE, Queens College CUNY, UCF, UCLA and ZEW Mannheim – for helpful comments. We gratefully acknowledge financial support from the ESRC (Transparency in Procurement, ES/N00776X/1) and The Spanish Ministry of Economy and Competitiveness (Grant ECO2014-59959-P).

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# 1 Introduction

Firms often compete with the same rivals in different, vertically connected, markets: say, upstream markets for inputs and downstream markets, where they sell their output. When these firms have significant market power, the resulting strategic interaction can become complex and closely dependent on market microstructure. In this paper, we take a fresh look at the deceptively straightforward scenario, where the input is provided competitively by a large number of small (capacity constrained) suppliers. Using a novel model of price determination, we show that the usual negative welfare effects of downstream market power are mitigated through a feedback channel to the upstream market that has generally been considered anticompetitive.

Our point of departure is the conventional wisdom that firms engaged in vertical multi-market competition have an incentive to foreclose: to reduce the rivals' production by somehow starving them of input. Models often fail to capture the full ramifications of this observation.<sup>1</sup> To illustrate this, assume for the moment that the – atomized – supply of input is completely elastic. It may then appear natural – though we will argue that perhaps incorrect – to model upstream competition as firms simultaneously choosing quantities (as the input price is “given”). It is then clear that a result of downstream market power is a lower output, resulting in a lower input demand. We contend that the incentive to increase one's (input) quantity in order to decrease the rivals' is seriously underestimated if we only take account of the strategic substitutability arising from quantity competition.

Modelling competition in this restricted way implies that a firm cannot directly affect its rivals' input levels. We believe that this is an unnecessary, and often unrealistic, constraint. To address this concern, we propose an alternative microstructure that explicitly takes into account the firms' ability and desire to foreclose.

The key feature of our model of the (upstream) market is that the same supplier may be approached by multiple potential buyers simultaneously, even though she is constrained to deal with at most one of them.

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<sup>1</sup>We present a detailed discussion of the related literature below.

By targeting specific suppliers, firms can affect whether their purchase is at the expense of their rivals. By directing their demand at the suppliers that the rivals intend to use, they can potentially reduce the rivals' input (and hence output). When every firm can engage in such “poaching” activity, the equilibrium strategies incorporate defensive tactics: *competitive foreclosure* – characterized by aggressive competition for a subset of suppliers – ensues.

We streamline bargaining by assuming that the firms make take-it-or-leave-it offers to the suppliers of their choice – or, equivalently, each (unit-)supplier holds a first-price auction.<sup>2</sup> Importantly, this mechanism is completely decentralized: each supplier decides independently who to deal with, given the prices offered to her by the firms – to which the latter are assumed to be fully committed.

A novel implication of competitive foreclosure relative to the traditional ‘raising rival’s cost’ scenario is that the higher price paid for input is accompanied by an *increased* input purchase. The intuition is transparent even before we specify the details of our model: traditional foreclosure is considered as an asymmetric situation, where Firm 1 forecloses Firm 2 by capturing the lower part of the supply function, thereby ostracizing its rival to its higher part. Faced with the higher cost, the foreclosed firm purchases less input. In our set-up, the firms are trying to foreclose *each other*. As a result of internalizing the externality, the willingness to pay for a unit of input increases for both firms, leading to a bidding war, resulting in both higher input prices and higher input volume (where the supply curve is shared symmetrically). Put another way, the higher price results from a shift in the demand curve, rather than a movement along it.

Our stylized model captures a wide range of scenarios. The inputs firms compete for may be materials (e.g., mineral ore), parts, or even machinery.<sup>3</sup> All we need is a large number of small – capacity constrained or exclusively dealing – potential

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<sup>2</sup>Our firms have market power, so they are likely to have bargaining power as well. Moreover, as we will see, giving all the bargaining power to them is not crucial as, due to competition, they will not be able to benefit from it.

<sup>3</sup>In fact, for our purposes it is irrelevant whether the firms add any value: they could be intermediaries.

suppliers. Perhaps the most obvious example of such input is labor.<sup>4</sup> But labor is far from being unique in this respect. Advertising space for advertisers (on web pages, in newspapers, in broadcasting breaks, etc.), or shows for pay-TV platforms are among the less ordinary examples of input markets where our discussion is relevant. When Netflix landed in Europe and was able to sign some of the successful TV dramas into its streaming service, local operators, like Canal Plus and Sky, felt it necessary to find ways, including banding together with operators in other markets, to outbid their new rival for some of the shows.<sup>5</sup> That willingness to pay for an input that would otherwise go to the competitor is higher is well known. What we argue in this paper is that the increased willingness to pay also results in more input purchased – and so in larger output in the final market. Continuing with our example, the same pressure from Netflix made BBC change its policy of signing new seasons only after previous ones had been aired (and so have been fully evaluated).<sup>6</sup>

The insights obtained apply to cost reducing inputs as well. After all, what is needed is that the foreclosed “input” increases *profits*, not necessarily output. For instance, this is the case if the input suppliers are scientists or labs that could provide process-innovations.<sup>7</sup> Hiring such a supplier (or buying their exclusive patent) at the expense of the rival is one way of ‘raising the rival’s cost’ – a known example of foreclosure – that, other things equal, increases the firm’s profits. The literature

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<sup>4</sup>The insights that we obtain are most relevant for labor markets with identifiable individuals like top management, academics, professionals, etc, where personalized deals are common.

<sup>5</sup>See "Netflix Global Growth Faces New Threats", in *The Wall Street Journal*, Jan 17, 2016. (On-line edition, consulted February 9th, 2017.)

<sup>6</sup>See "How European Networks Are Trying to Fight Off Streaming Rivals", *The Hollywood Reporter*, online version September 8, 2016, (consulted on February 9th, 2017). The entry of Netflix and other operators, like Amazon, into local markets was linked to a sharp increase in drama production. As the agent Pascal Breton put it in 2016, “... all these players in Europe will battle to buy the best European shows. In the next three years, the market will grow dramatically, probably double.” (*Variety*, October 15th, 2016, on-line edition consulted February 9th, 2017.)

<sup>7</sup>Following the interest shown by Facebook in the Israeli startup Waze, Google managed to bid and acquire the company whose main expertise was using satellite pictures to generate maps and traffic data. Peter Cohan, an analyst of the startup market, wrote in <http://www.forbes.com> that “(a)ccording to Ha’aretz, “Google’s interest in Waze stems principally from its aim of blocking Facebook’s growth.”” (June 9th, 2013; online version consulted on February 9th, 2017.)

on R&D has long recognized the strategic incentives associated with R&D through the effect that a reduction in costs has on output market behavior and so on profits.<sup>8</sup> What we add to this literature is an additional strategic incentive, this time associated with the direct competition for R&D assets.

We capture this wide variety of applications via our simultaneous bidding model in the input market and a reduced form model of the output market, which simply maps the realized input purchases into revenues for the firms.

Our first result confirms the intuition that competition drives out price discrimination: in equilibrium, even with targeted offers and the suppliers' reservation values being public, all traded units sell at the same price. Next, we show that in the benchmark case, where the firms do not affect each other in the product market,<sup>9</sup> the unique equilibrium of our targeted-offer input market institution leads to the competitive outcome (in that market). When firms do have downstream market power over each other, the market-clearing equilibrium is different. In it, firms target the same suppliers. The resulting willingness to pay exceeds the competitive one as it includes a "conjectural variation" of  $-1$ : being chosen by a supplier over a rival not only increases a firm's input by one unit, but – relative to the counterfactual – it automatically decreases the rival's input by one unit as well. Due to this increase in the firms' input demand, both – the still common – input price and, by market-clearing, the aggregate input increase.

This is not the only equilibrium when externalities are present. However, all other equilibria exhibit supra-competitive levels of aggregate input as well – together with even higher input prices than the market-clearing equilibrium. Consequently,

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<sup>8</sup>This effect is at the heart of the literature on the relationship between competition and the incentives to innovate, in the tradition of Loury (1979) and Lee and Wilde (1980). (See a survey of this early literature in Reinganum, 1989.) Also, this is the effect that plays an important role in the literature that directly addresses the interaction between the incentives to pursue R&D and to foreclose rivals in the traditional vertical relations literature. (See, for instance, Stefanides, 1997, Banerjee and Lin, 2003, and Chen and Sappington, 2011.)

<sup>9</sup>This could be because they sell in different markets (either in physical or in product space) or because the downstream market is competitive.

these additional equilibria are characterized by excess supply: there are suppliers that do not trade who would accept less than the market price. These equilibria arise from the coordination-game nature of the interaction: as firms want to make offers to the same suppliers, if the other firms stop targeting a few suppliers it is a best response not to make an offer to them either. Buying from them would no longer have the added value of reducing a rival's supply. It is this wedge between the value of a unit of input that would otherwise be employed by a rival and one that would not be, that explains the possibility of prices above (marginal) reservation prices.<sup>10</sup>

When the input in question is labor, these additional equilibria are actually characterized by unemployment. Note that the rationale behind wages above reservation wages is quite different from the rationales discussed in the literature on efficiency wages. Here it is not the productivity of the worker that is incentivised by the wage, rather an output market externality is added to the value of that productivity, when the offer is to a contested worker.

In markets for unskilled labor, it may be argued that our pricing model, which posits personalized offers, is less realistic. If we assume that firms cannot target their offers, for instance, because workers are “anonymous”, we obtain as the only equilibrium the full-employment one. Nevertheless, we show that, introducing dynamics to the anonymous model – together with the possibility of using employment in one firm as a “label” that allows imperfect targeting – may reestablish equilibria with unemployment.

To complement the abstract nature of the preceding analysis, we fully develop a model of the output market with differentiated products and constant returns, that fits the reduced form model that we use in the analysis. We also use simple models to demonstrate that our results go beyond competition for inputs, however broadly construed. Indeed, noting that the crucial characteristic we need is that

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<sup>10</sup>We assume that product demand and input supply are generated by different people. This is in contrast with general equilibrium (macro) models – where consumers and workers are the same – that display multiple equilibria and unemployment via different rational expectations equilibria (see Silvestre, 1993, for an excellent survey).

the market where competitive foreclosure arises closes before its vertically related counterpart, we discuss two alternative vertical structures, competition for retailers and competition for production orders, where the same insights apply.

The paper is organized as follows. The next section reviews the many different strands of the related literature. Section 3 presents the model of input market competition and the (reduced form) model of product market revenues. In Section 4, we analyze the benchmark case where a firm's supply decision does not affect the other firms' revenues, and obtain the competitive (input market) outcome. Section 5 then introduces output market interaction, obtains the new market-clearing equilibrium, and characterizes the rest of the equilibria. In Section 6 we discuss how the results change when firms cannot target their wage offers. Section 7 illustrates our main model with micro-founded revenue functions, and Section 8 discusses how our ideas apply to alternative vertical structures. Finally, Section 9 concludes. All proofs are in the Appendix.

## 2 Related literatures

There is an extensive literature on foreclosure (see Rey and Tirole, 2007, for an excellent survey), mainly concerned with vertical contracting. In that literature, downstream firms with market power also have an incentive to lure upstream firms into contracts that make it harder for rivals to obtain their inputs. The focus there is on whether these incentives are stronger than upstream firms' own incentives not to enter into these deals, or on the contractual forms that may affect competition to the contracting parties' advantage. Instead, we have in mind an upstream market where suppliers do not possess market power, and cannot sensibly deal with more than one buyer. Foreclosure here appears through quantity "purchases", rather than through – from a competition law point of view possibly problematic – vertical restraints. Even more importantly, the literature on foreclosure compares a *fait accompli* with the absence of foreclosure, while our model emphasizes the very process of competing to foreclose, which need not lead to an overall winner.



Also closely related, is a strand of the literature on auctions with downstream externalities (see, e.g. Jehiel and Moldovanu, 2000). In this tradition, Mayo and Sappington (2016) explicitly tackle the foreclosure incentives of rivals bidding for a unit of input, and study conditions for the outcome of the auction to be efficient. These papers do capture the incentives to foreclose in a bidding context, in scenarios with a single overall winner, not unlike the literature on foreclosure mentioned above. We take these incentives to a more nuanced scenario where winners in one battle do not emerge as winners in the war: competitive foreclosure may lead to a symmetric outcome. As a result, our model allows us to analyze the effects of foreclosure incentives not just at the margin (and hence on prices) but also on quantities, both in the input and the output market, an issue that by design is absent in their – and the literature’s – analysis.

A few papers have modeled the interaction between generic input and output markets when firms have market power in both.<sup>11</sup> Stahl (1988) is among the first to analyze the effect on output market outcomes of intermediaries’ competition for upstream inputs. Intermediaries compete in prices for inputs that then are sold downstream. Price competition leads to no excess supply, by definition: all supply at the posted price is assumed to be taken by the intermediary making the offer. When ties are broken in a particular way (one winner takes all, even when tying), then the output price may be larger than Walrasian, but single-price, market clearing in the input market is always guaranteed. In his quest to analyze the effects of vertical mergers, Salinger (1988) proposes a model where there is Cournot competition between the sellers in both markets. As firms are assumed price takers in the input market, his model is the ideal straw man for us. Spulber (1996) has price setting intermediaries, but he too assumes away externalities. Yanelle (1997) studies a model of bank competition that shares some interesting features with the present one. She also obtains that there is a range of equilibria, at different prices (rates) for funds. When banks and borrowers (entrepreneurs) compete with each other for funds, lenders face a coordination problem: borrowers and banks can fulfill

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<sup>11</sup>The literature on inter-market interactions when firms act in multiple output markets is abundant. A classical reference here is Bulow et al. (1985).

their offers if they get sufficient other lenders on board. Thus multiplicity ensues. We also obtain multiple equilibria for inputs as the result of a coordination problem, but in our case input buyers, not their sellers, are the ones that face this problem. Competition in the output market is the origin of the externality in the input market. More recently, Eső et al. (2010) discuss quantity competition in this same setting, but assume exogenous (inelastic) supply of input and efficient allocation of this input to firms. The insight that market power may – partially – defeat itself through the linkage between the two vertically related markets, a central tenet in our analysis, is absent in these papers.

Our pricing model in the input market is also related to McAfee (1993), Peters and Severinov (1997), Burguet and Sákovics (1999), Julien et al. (2000), and De Fraja and Sákovics (2001), among others.<sup>12</sup> All these papers consider similar institutions involving personalized offers, with the main difference that they all assume that each buyer can only participate in a single auction. Thus, price differentials (intra- or inter-firm) are a consequence of different realizations of (mixed-strategy) equilibrium participation in these auctions. In contrast in our model, firms are allowed to make (and required to honor) offers at several auctions simultaneously.

Note that our model is distinct from those that also model competing “retailers” bidding for input but consider the supplier as a monopolist (Marx and Shaffer, 2007, 2016, Miklos-Thal et al., 2011, Rey and Whinston, 2013). In those papers, each retailer has a single possible contract, so foreclosure equates exclusion. Also the retailers are concerned with reaping the industry monopoly profit in equilibrium, which is far from feasible in our model. If we considered a trade union responding to the offers of the retailers, we would not have the internalization of the external effect, which is the basis for our result.

Our insights relate to many specialized fields. As a taster, let us discuss some of

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<sup>12</sup>Our model could be interpreted as each supplier auctioning off her services. Lang and Rosenthal (1991) study a – finite, discrete – model of bid-competition very similar to the one we use in this paper, except that bids cannot be targeted. Pepall and Richards (2001) have a single star supplier (plus a price taking fringe) for whom firms bid. Palomino and Sákovics (2004) have clubs bidding for high and mid-level talent.

the most relevant, related labor literature. Bhaskar et al. (2002) – and the references there – provide abundant evidence of wages above reservation values, as some of the equilibria of our model exhibit. Of course, efficiency wage models rank high as explanations of this phenomenon.<sup>13</sup> Here, we have assumed away all traditional motives behind the rationale for efficiency wages. Related to this paper, Shy and Stenbacka (2015), allow for different wage offers to incumbent and poached workers. Their motivation is the profit and welfare implications of anti-poaching policies when switching workers are affected by both a productivity change and a – worker specific – cost of switching. Switchers may obtain wages above the wages of stayers, so that wage differentials result from switching frictions (which we do not have). Other studies have related product market imperfections with unemployment, when labor (union) has bargaining power in wage negotiations.<sup>14</sup> This bargaining power allows suppliers to capture part of firms’ rents in the output market, and so drive wages above reservation wages. In our model, suppliers are price takers. (In fact, our model has equilibria where suppliers appropriate no rents.) Wages are above reservation wages as a consequence of firms’ attempts to capture competitors’ rents. Kaas and Madden (2004) also analyze the feedback between product and input market power, and also obtain the possibility of unemployment as an equilibrium outcome. Firms first post wages and then, after observing all choices, announce a maximum amount of labor they are willing to hire at their posted wage and, possibly, rationing follows. This two-stage competition for labor allows high wages to be equilibrium: any deviation downwards triggers a punishment by other firms in the form of large demands of labor that drives the deviator out of the market. These punishments themselves are sustained by the threat to a failing “punisher” of being also driven out of the market. Thus, in a sense unemployment is the result of collusion among firms, with collusion-type mechanisms to sustain it.<sup>15</sup> We do not need this particular, two-stage model of the input markets or the endogenous price

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<sup>13</sup>See, for instance, Yellen (1984).

<sup>14</sup>See Blanchard and Giavazzi (2003). Koskela and Stenbacka (2012) and Booth (2014) are recent examples of this literature.

<sup>15</sup>They require to have at least three firms competing, so that each deviant is disciplined by (at least) two punishers, each one disciplining each other.

rigidities that it postulates at the time contracts are offered. Rather, high prices are the consequence of firms fiercely competing with alternative buyers that are also output market competitors.

### 3 The base model

There are two identical firms competing both in the input and the product markets. To avoid complications arising from indivisibilities, we assume that there are a continuum of unit-suppliers of a homogeneous input, indexed by  $z \in [0, 1]$ . Their – exogenous and common knowledge – reservation prices imply an aggregate supply function, denoted by  $t = S^{-1}(p)$ , w.l.o.g. assumed to be weakly increasing in price,  $p$ , so that the inverse aggregate supply function<sup>16</sup> is  $S(t)$ .

The input market operates as follows: each Firm  $i$  sets a price schedule,  $P_i(z)$ ,  $z \in [0, 1]$ , specifying a personal price offer to each supplier.<sup>17</sup> Firms are committed to their offers: if an offer is accepted the firm must honor it in all contingencies.<sup>18</sup> Let  $\mathbf{P}$  denote the full profile of price schedules. The suppliers' decisions are simple: they observe their offers, and accept (one of) the highest if it is no less than their reservation price. Given  $\mathbf{P}$  and the acceptance strategies of the suppliers,<sup>19</sup> we let the measure of suppliers enlisted by Firm  $i$  be  $t_i(\mathbf{P})$  and the total cost for Firm  $i$  be  $b_i(\mathbf{P})$ .<sup>20</sup>

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<sup>16</sup>In the tradition of economics, we allow for vertical segments in the inverse supply *correspondence*.

<sup>17</sup>If a firm does not make an offer to some suppliers, we simply set the price offered to zero.

<sup>18</sup>Off the equilibrium path this may require a firm to purchase more input than it needs, though it still has the option not to utilize it for production.

<sup>19</sup>For simplicity, we suppress the supplier strategies in the notation.

<sup>20</sup>By analogy to the Law of Large Numbers, we assume that these expected values exist and coincide with the realized values with probability 1. For example, letting  $\nu_i^1(p; P)$  and  $\nu^2(p; P)$  represent the measures of suppliers who receive a highest acceptable offer  $p$  only from Firm  $i$  and from both firms, respectively, and assuming indifferent suppliers mix fifty-fifty

$$t_i(\mathbf{P}) = \int_0^\infty \left[ \frac{\nu^2(p; \mathbf{P})}{2} + \nu_i^1(p; \mathbf{P}) \right] dp,$$

Once the inputs are purchased, the firms produce. To maximize generality, we do not model the production process and the product market competition in detail. We simply assume that given any amount of input bought by the rival firm,  $t_r$ , each firm's net revenue as a function of its own input level,  $t_o$ , is  $R(t_o, t_r)$ . That is, a firm's payoff, given the acceptance strategies of the suppliers and the vector of price schedules  $\mathbf{P}$ , is

$$R(t_o(\mathbf{P}), t_r(\mathbf{P})) - b_o(\mathbf{P}).$$

It is natural to assume that  $R$  is (weakly) increasing in  $t_o$  and (weakly) decreasing in  $t_r$ , for example, due to free disposal. On the other hand, for some standard models of market competition, this payoff function is not concave. Thus, we make alternative assumptions on  $R$  instead. Below, the subscripts of  $R$  represent partial derivatives with respect to the corresponding variable.

**Assumption 1**  $R_1(t_o, t_r) \geq 0$  and  $R_2(t_o, t_r) \leq 0$ . Moreover, i)  $R_{11}(t_o, t_r) < 0$ , ii)  $R_{12}(t_o, t_r) \leq 0$ , and iii)  $R_{11}(t_o, t_r) - 2R_{12}(t_o, t_r) + R_{22}(t_o, t_r) \leq 0$ .

Assumption 1iii) implies that a firm's revenue is concave in the amount of input that it poaches from the rival firm. This is a sufficient condition that allows us to characterize the set of symmetric, pure strategy equilibria using the first-order approach. As we will see in Section 7, typical Cournot or Dixit models satisfy this assumption.

As we mentioned in the Introduction, the input may reduce the production cost, instead of increasing output. For instance, suppose that  $t$  represents the number of scientists working in the process-innovation division of the firm, and model this number as a continuum.<sup>21</sup> Suppose that the firm's (constant) marginal cost of production is a decreasing function of this number,  $c_i(t_i)$ . Then, we recover our

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and

$$b_i(\mathbf{P}) = \int_0^\infty p \left[ \frac{\nu^2(p; \mathbf{P})}{2} + \nu_i^1(p; \mathbf{P}) \right] dp.$$

<sup>21</sup>Likewise,  $t$  could represent the number of patents of cost-reducing innovations up for (exclusive) contracting.

basic model letting

$$R_i(t_o, t_r) = \pi_i(c_i(t_o), c_j(t_r)),$$

where  $\pi_i$  is the profit of firm  $i$  net of R&D-division salaries. For instance, if firms produce homogeneous goods and compete downstream in quantities, then  $\pi_i$  is the Cournot profit function.<sup>22</sup>

As a final preliminary, let us present a result that shapes the nature of equilibrium strategies.

**Lemma 1** *In any equilibrium where both firms are active, the law of one price holds.*

In other words, despite the flexibility afforded by the model to offer different prices to different suppliers, all the accepted offers in equilibrium must be at a common price.<sup>23</sup> Note that this does not imply that we could prescind with personalized offers, as the feasibility of coordinating offers on the same suppliers matters. Additionally, personalized offers play an important role as possible deviations, reducing the number of equilibria. We will discuss these issues in more detail below.

## 4 The competitive benchmark: no externality

We start our analysis by looking at the benchmark situation, where there is no strategic interaction in the product market (that is,  $R_2(t_o, t_r) = 0$ ) – for example, because the firms sell in different markets. In this case, the partial derivative of net revenue with respect to own input,  $R_1(t_o, t_r)$ , captures a firm’s marginal willingness to pay for input when it has already bought  $t_o$  units. Therefore, the market clearing – or competitive – outcome is defined by

$$R_1(t^c, t^c) = S(2t^c) = p^c. \tag{1}$$

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<sup>22</sup>It is easy to construct linear-demand, logistic-shaped (in  $t$ ) marginal cost examples that also satisfy Assumption 1.

<sup>23</sup>This result need not generalize to more firms. For example, a four firm market does have an equilibrium where two different prices are paid.

**Remark 1** It is important to note that, if the firms were input price takers, the market-clearing outcome would still be determined by (1) even when firms interact in the product market: if a firm cannot affect the amount/price of input its rival buys then it does not matter – for the firm’s behavior in the input market – whether its revenues depend on the rival’s input level.

**Remark 2** A further relevant observation is that this competitive input market equilibrium is only competitive in a partial equilibrium sense. Firms’ willingness to pay for input does not equal price times marginal product of input when the product market is not competitive. The input market clears and the price equals firms’ willingness to pay but less input is bought than in the fully competitive scenario (c.f. Stahl, 1988).

The following result shows that our price setting game uniquely implements the competitive outcome:

**Proposition 1** *In the absence of downstream interaction, the unique equilibrium outcome is market clearing: all input is bought at the competitive price,  $p^c$ , and firms purchase their competitive input demand,  $t^c$ .*

In the symmetric equilibrium,<sup>24</sup> each firm offers  $p^c$  to the same set of suppliers, comprising *twice* the amount of input it wishes to buy at this price so that each supplier willing to accept  $p^c$  receives two offers and accepts either of them with equal probability. Note that the fifty-fifty mixing is endogenously derived, not assumed.<sup>25</sup> If the supply function is strictly increasing, the symmetric equilibrium strategy is equivalent to naming a posted price, as the equilibrium clears the market.<sup>26</sup> Given this, no firm would like to attract another supplier as it would have to pay a price

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<sup>24</sup>Of course, there are asymmetric equilibria – where both firms and suppliers use heterogeneous strategies – as well, leading to the same outcome.

<sup>25</sup>Proposition 1 – as well as our subsequent results – generalizes to asymmetric firms, but in that case the unique (supplier-symmetric) equilibrium requires the suppliers to use a different mixed strategy. See Proposition 1 in Burguet and Sákovic (2017).

<sup>26</sup>With fully elastic supply ( $S(t) \equiv r$ ) the argument is slightly different but the equilibria look the same.

above  $p^c$ , the firms' marginal willingness to pay. Similarly, no firm would profit from shedding a supplier, since their marginal product equals  $p^c$ . Finally, there is no way to buy from any supplier cheaper than for  $p^c$  either, confirming the equilibrium. Uniqueness can be established by observing that in equilibrium, practically by definition, there cannot be a positive measure of input bought for any price other than  $p^c$ .

This last observation explains why firms coordinate on making offers to the same set of suppliers. In equilibrium, any supplier,  $z$ , who only received a single offer would have to be paid her reservation price. But if this price were lower than some price paid by the rival, then the rival would profit from not buying from that supplier and buying from  $z$  for  $\varepsilon$  more than her reservation price instead. As a result, competition drives out price discrimination, despite reservation prices being common knowledge.

## 5 Equilibria with externalities

We now investigate the effects of (shared, product) market power on the equilibrium outcome in the input market by assuming that  $R_2(t_o, t_r) < 0$ .<sup>27</sup> This externality does not only imply that the revenue of a firm depends (negatively) on the input level of the rival. It has an additional – more intricate – consequence. When we totally differentiate  $R$  with respect to  $t_o$  we obtain

$$\frac{dR(t_o, t_r)}{dt_o} = R_1(t_o, t_r) + \frac{\Delta t_r}{\Delta t_o} R_2(t_o, t_r).$$

Thus, due to the externality,  $\frac{\Delta t_r}{\Delta t_o}$ , the effect an infinitesimal change in a firm's own input level has on its rival's (holding the rival's *strategy* constant), becomes relevant. The crucial observation is that this effect is determined endogenously. Buying from an additional supplier has a different effect on a firm's revenue depending on whether the supplier would have sold to the rival firm – the supplier is *contested* – or not.

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<sup>27</sup>Note that we are implicitly assuming that (in equilibrium) the firms are using for production all their purchased input. While there are situations where this is not the case, the gain in generality is not worth the increase in complexity and the loss in focus.



In the former case  $\frac{\Delta t_r}{\Delta t_o} = -1$ , while in the latter  $\frac{\Delta t_r}{\Delta t_o} = 0$ . Therefore, the marginal value of attracting a supplier away from a rival is the sum of two (positive) effects:  $R_1(t_o, t_r)$  and  $-R_2(t_o, t_r)$ . At the same time, the marginal value of attracting an uncontested supplier is simply the increase in net revenue made possible by using the extra input,  $R_1(t_o, t_r)$  – just as in the benchmark case. This increase is lower than when obtaining the input from a contested supplier, as it does not include the increase in revenues caused by the reduction of input – and thus output – of the competitor. As we will see, in equilibrium each firm will seek to make offers to the same set of suppliers as its rival – resulting in all contracted suppliers being contested – and consequently they have the higher marginal willingness to pay for all of them.

**Proposition 2** *When  $R_2(t_o, t_r) < 0$ , the competitive outcome  $(t^c, p^c)$  is no longer supportable in equilibrium. Nonetheless, there exists a unique market-clearing equilibrium outcome, with each firm buying from  $t^*$  suppliers, paying them the same price  $p^*$ , where  $(t^*, p^*)$  solve*

$$R_1(t^*, t^*) - R_2(t^*, t^*) = S(2t^*) = p^*. \quad (2)$$

Note that  $t^*$  and  $p^*$  are the market-clearing input level and price, when the two firms make offers to the same set of suppliers. When each supplier is contested, decreasing  $t_o$  will increase  $t_r$  by the same amount. Similarly, the best way to increase  $t_o$  is to lure some of the suppliers away from the rival. Thus, indeed the marginal valuation is given by  $R_1(t_o, t_r) - R_2(t_o, t_r)$ . Market clearing then follows from similar arguments as in the benchmark model.

To see why the competitive outcome is not an equilibrium, simply observe that  $R_1(t^c, t^c) - R_2(t^c, t^c) = p^c - R_2(t^c, t^c) > p^c$ , so the firms have an incentive to outbid their rival for some of the suppliers that would sell to its rival with positive probability. The following corollary is also immediate.

**Corollary 1**  *$p^* \geq p^c$  and, therefore,  $t^* \geq t^c$ . Moreover, the first inequality is strict unless supply is infinitely elastic, while the second is strict unless it is fully inelastic.*

In other words, the firms' market power in the product market leads to higher prices and increased input purchase. Thus, somewhat surprisingly, imperfect competition downstream leads to a more competitive outcome than the “competitive” benchmark. The size of this effect increases with the elasticity of supply and the size of the production externality.

**Remark 3** Our unique *market-clearing* equilibrium outcome could be implemented as one of the many market-clearing equilibria of a posted-price model, where the firms are required to buy from any supplier that accepts the price. However, the intuition behind all those equilibria is completely different, it has nothing to do with the externalities, rather the diminished incentives to outbid a rival, as that would lead to all the (willing) suppliers switching to accept the firm's offer. (c.f. Dastidar, 1995). Tellingly, the competitive equilibrium outcome of Proposition 1 is also one of the posted-price equilibria.

Proposition 2 characterizes a focal equilibrium with supra-competitive prices and quantities. Typically, there exist other equilibria as well, but they lead to even higher prices and consequently to excess supply: some suppliers who would be willing to trade at the market price are not approached by either firm.

**Proposition 3** *Unless either<sup>28</sup>  $R(t^*, t^*) - t^* p^* = 0$  or the elasticity of supply is zero, there exists  $\underline{t} < t^*$  such that for each  $t \in [\underline{t}, t^*]$ , there exists an equilibrium where both firms offer*

$$p = R_1(t, t) - R_2(t, t) \geq S(2t) \quad (3)$$

*to (the same)  $2t$  suppliers among those with reservation price below  $p$ , and make no acceptable offer to the rest. Moreover, all symmetric equilibria must be characterized by such behavior for some  $t \in (t^c, t^*)$ .*

As we have seen in Proposition 2, when  $2t^*$  units of input are purchased, the market price equals the marginal supplier's reservation price, as well as the marginal

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<sup>28</sup>Non-negativity is guaranteed by Assumption 1iii. Given that we are studying firms with (downstream) market power, one might expect positive profits. However, as we will see in Section 7, it need not be the case in extremely simple and standard models. That is, input competition may eliminate oligopoly rents.

willingness to pay of the firms. On the other hand, for  $t < t^*$ , the inequality in (3) is generically strict and thus we have excess supply. The root cause for this departure from market clearing is yet again the externality we have been discussing. A supplier who would be willing to accept the equilibrium price may not receive an offer from Firm 1 for the mere fact that she does not receive an offer from Firm 2. This situation arises when Firm 1 is already buying from sufficient suppliers so that  $p > R_1(t, t)$ . That is why we have  $R_1(t, \underline{t}) \leq S(2\underline{t})$  – and, therefore,  $t^c \leq \underline{t}$ . See Figure 1.<sup>29</sup>

INSERT FIGURE 1 HERE

**Remark 4** One might think that, when the firms see the availability of supply at lower prices, they would wish to buy some if only they had the option of a second round of offers. This, however, is not the case. The firms can predict exactly how much supply they will purchase in equilibrium, so if they wished to make acceptable offers for additional units, they could have done so to start with. We will return to the potential relevance of dynamics in the next section.

The extent of multiplicity, and thus the maximum distance of equilibrium outcomes from the market-clearing price (upwards) and quantity (downwards), positively depends on the elasticity of supply and on the size of the production externality – which is likely to decrease with the number of firms.

Thus, market power in the product market may create segmented competition for otherwise homogeneous suppliers. Suppliers targeted by rival firms become more

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<sup>29</sup>As we have mentioned, we need Assumption 1 to guarantee that a symmetric equilibrium exists, not that an equilibrium with excess supply exists. Indeed, concavity of the revenue function in the flow of input from the rival firm has two implications that count here. Firstly, concavity guarantees second order conditions for the strategies identified by a first-order approach. This first-order approach always identifies candidate symmetric equilibria with and without excess supply. If the revenue function is  $C^2$ , strict local concavity at a candidate equilibrium with market clearing implies strict concavity for neighboring candidate equilibria with excess supply. Secondly, concavity guarantees that a local “best response” (maximum) is also a global one. Absent concavity, the first-order approach is not appropriate, whether we refer to an equilibrium with or without excess supply.

valuable than untargeted suppliers. A type of input for which excess supply – unemployment – has been widely discussed is labor. Proposition 3 advances a novel, additional explanation for this phenomenon.

In a market for unskilled labor, our model of pricing competition, which includes the possibility of targeting wage offers, is not very plausible, particularly when firms are not large in that market. For this reason, in the next section we discuss how our insights are affected when we exclude the possibility of targeting offers at particular suppliers.

## 6 Anonymous suppliers

Assume that firms can decide how many offers they make and at what prices, but cannot address particular offers to particular suppliers, thereby rendering (perfect) coordination impossible. Thus, even in equilibrium, firms can only have a probabilistic idea of whether the suppliers receiving their offers are contested or not. As we will see, this modification of our model eliminates the equilibria with excess supply of the previous section, while it retains the non-competitive market-clearing equilibrium as the unique equilibrium where the law of one price applies.<sup>30</sup>

Suppose that the total mass of input is  $T$ .<sup>31</sup>

**Proposition 4** *Under supplier anonymity, in the only (firms') pure-strategy, symmetric equilibrium with a single price, the non-competitive market-clearing outcome,  $(t^*, p^*)$ , emerges.*

The intuition behind the result is simple: if both firms send offers with the same price, a firm attracts any supplier that gets no other offer plus a portion of the suppliers that get the competing firm's offer. By sending offers with slightly higher

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<sup>30</sup>As this is but a robustness check – under anonymity – we do not investigate the existence of equilibria where there can be input bought at different prices.

<sup>31</sup>Until now, there has been no need to specify the total mass of input, as the “tail end” of the supply curve played no role: those suppliers would never be targeted.

prices, the firm may increase (discrete jump) the conditional probability that an attracted supplier has received an offer from the rival, unless all the suppliers do get one. Thus, the only possible equilibrium must have all firms sending offers to all suppliers, in which case the reservation price of any non-enlisted supplier must be no less than the market input price. That is, we must have market clearing. When all the suppliers receive an (identical) offer, anonymity becomes irrelevant: firms know that all the suppliers are contested, so their willingness to pay is the same as under targetability.

**Remark 5** It is worth emphasizing that a higher price cannot be an equilibrium outcome, not because this would give firms incentives to pay lower prices and still attract (uncontested) suppliers, but rather the opposite: firms would have incentives to increase their offers and so compete more fiercely for (some) suppliers.

This observation indicates that the high-price equilibria of the model with targeting are not artifacts of the inability of firms to react to competing firms' offers (by enlisting uncontested suppliers). In fact, as we show next, if we allow "some" dynamics in our model we can reconcile supplier anonymity with equilibrium excess supply.

The dynamic model is based on the observation that – even under anonymity – the mere fact of supplying a rival firm constitutes a label that may be used to target a group of suppliers (c.f. Shy and Stenbacka, 2015). If firms have the opportunity to react to rival offers by targeting new offers to the rival's suppliers, then – as we will show – the possibility of endogenously segmented input markets reemerges.

Let us maintain the assumption that offers are binding for firms, but suppliers can freely walk out of a contract. Also, suppose that, before production takes place but after all initial offers have been made and the suppliers have responded, a randomly selected firm has the opportunity to send new offers. This time, the selected firm can target (as a group) a subset of the suppliers that have tentatively accepted an offer from the rival firm.<sup>32</sup> Both assumptions are sensible if, for instance, the input

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<sup>32</sup>That is, anonymity is maintained within the group of suppliers enlisted by the rival and the firm can choose how many offers to send within that group.

is labor.

In addition to Assumption 1, let us assume that  $R_{22} > 0$ . Also, to simplify the analysis, let us assume that each firm makes all offers with the same price, although different firms may choose different prices to offer. Further, to avoid repetition,<sup>33</sup> we assume that in the second period only the suppliers hired in the first period may receive offers.

**Proposition 5** *In the anonymous game, where each firm is equally likely to have an (exclusive) chance to target new offers at some proportion of the suppliers enlisted by the rival, there exist equilibria where firms buy input  $t$  each at price  $p$ , where  $p > S(2t)$ . That is, there are equilibria with excess supply.*

To understand the intuition behind this result, recall that excess supply equilibria were not possible without a second move because a firm had incentives to deviate and make offers to fewer suppliers with a slightly higher price, so that it ended up buying the same amount of input as before. By doing so, the firm would significantly reduce the input bought by the rival. Hence, that deviation would result in lower aggregate input whether or not there were a second period. Now, if the other firm has an opportunity to react, which occurs with 50% probability, it will basically “flip” the input distribution, so that it would be the deviating rival who would suffer the loss. The joint profits of the two firms would drop, and (in the most favorable case), the expected profits of the deviating firm would still only be 50% of those joint profits.

## 7 Microstructure: the output market

In the preceding sections, we have used a reduced-form approach to modelling production and product market competition, positing general revenue functions. Here,

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<sup>33</sup>Given the analysis previously displayed in this paper, it should be clear that this feature would be part of an equilibrium anyway.

we display a worked out example based on Dixit's (1979) quantity competition model with differentiated goods, which in the limit coincides with a linear Cournot model.

Thus, assume that the representative consumer has utility function  $U(m, q_1, q_2) = m + \sum_i (aq_i - \frac{b}{2}q_i^2 - cq_1q_2)$ , where  $q_i$  represents the (quality or) quantity of output of Firm  $i$ , and  $m$  is the rest of consumption – numeraire. The resulting inverse demand for Firm  $i$ 's product/service is

$$p^i(q_i, q_j) = a - bq_i - cq_j.$$

We also assume that firms operate constant (normalized to unit) returns to scale technologies.<sup>34</sup> Then, their net revenues will be

$$R^i(t_i, t_j) = (a - bt_i - ct_j)t_i.$$

Thus, Assumption 1 is satisfied, as in this model  $R_{11}(t_1, t_2) = -2b < 0$ ,  $R_{12}(t_1, t_2) = -c < 0$ , and  $R_{11}(t_1, t_2) - 2R_{12}(t_1, t_2) + R_{22}(t_1, t_2) = -b + c$ , which is negative as long as the price of each good is more responsive to changes in its own quantity/quality than to changes in the quantity/quality of the other good.

We may now compute the set of all symmetric equilibria that we have discussed in previous sections. For simplicity, assume the supply of input is infinitely elastic:  $S(t) = r < a$  for all  $t > 0$ . Then, the market-clearing equilibrium, discussed in Proposition 2 and characterized by  $R_1(t^*, t^*) - R_2(t^*, t^*) = a - 2bt^* = r$  leads to both firms purchasing

$$t^* = \frac{a - r}{2b}$$

units of input. Meanwhile, the – upstream – competitive purchase of input would be

$$t^c = \frac{a - r}{2b + c}.$$

That is the level at which  $R_1(t, t) = a - 2bt - ct = r$ . Note that indeed  $t^c < t^*$ .

We can now obtain the set of symmetric equilibria that Proposition 3 refers to.

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<sup>34</sup>This set-up is particularly well suited to analyze the competition between service providers, like consultancy companies, investment banks, security firms, cleaning services providers and even universities.

**Corollary 2** *In the linear Dixit model with constant returns to scale and infinitely elastic input supply, the set of symmetric-equilibrium input levels is  $[\underline{t}, t^*]$ , with the corresponding price  $a - 2bt \geq r$  for any given  $t$ , where*

$$\underline{t} = \frac{a - r}{2b + c - \frac{c^2}{2b - c}}.$$

Note that  $t^c < \underline{t}$ , so all equilibria involve a higher than competitive volume of input purchases. At the same time, for all equilibria except the market-clearing one, this higher purchase of input is accompanied by a higher input price ( $p = a - 2bt > r$ ). Thus, there exist suppliers who would accept a price strictly below the market price but don't get an offer. If labor is the input, these suppliers – workers – are “unemployed” – in this industry.

It is straightforward to see that the efficient (i.e., total surplus maximizing, given service demand and input supply) input and output (per firm) would be  $\bar{t} = \frac{a-r}{b+c} > t^*$ . That is, the most efficient of all the equilibria is the market clearing one,  $t^*$ . Moreover, in this model total industry profits –  $2(R(t, t) - pt)$  – are aligned with total surplus: they are increasing in  $t$  as well.

Let us consider the equilibrium set as we vary the degree of product differentiation. The market clearing equilibrium level of input is unaffected by  $c$ , while both the efficient (and profit maximizing) level of input and the competitive one are decreasing in the level of differentiation. The lowest possible level of input is U-shaped, while the corresponding maximum is constant. This is displayed in Figure 2.

INSERT FIGURE 2 HERE

Of particular interest is the limit, as  $c \rightarrow b$ . That limit case is the homogeneous-product, Cournot competition. We obtain as a limit  $\underline{t} = t^* = \bar{t} = \frac{a-r}{2b} > \frac{a-r}{3b} = t^c$ , with input price equal to  $r$  (and firms making zero profit). Thus, considering the feedback across markets, market power in the output market is completely bid away in the input market, and so efficiency is fully restored in this particular case.



## 8 Alternative vertical structures

Up to now, we have studied competitive foreclosure focussing on input markets – even if the nature of the input could be broadly construed. This seems the natural environment as the relevant feature of the extensive form is that the market exposed to foreclosure operates prior to the “other” market. Nonetheless, there are alternative scenarios where we can have competitive foreclosure in one market before another market clears – and, therefore, our model and its implications apply with only minor modifications. Let us discuss two salient such vertical structures here.

### 8.1 Competition for contracts/orders

While in terms of production inputs must precede outputs, when we talk about contractual commitments this need not be the case. In fact, it is common practice – for example, Tesla sells its new models through a preorder system – to secure orders for the finished product before production and even prior to purchasing the inputs.

To see how our model can be adapted to this scenario, suppose that our firms bid for unit contracts to their clients first. They do so as in our personalized pricing model: each firm simultaneously offers a contract (a price) to as many clients as they choose, possibly different ones for different clients. Each client then selects an offer to accept, if any. After signing these contracts, the firms obtain the necessary inputs in a market where they have market power. That is, they are oligopsonists in this input market. The cost function for each firm would then depend on the input purchases of both firms,  $C(t_o, t_r)$ . For instance, if the input – inverse – supply function is  $Z(t)$ , technology is constant returns to scale with only one input – one unit of input for each unit of output –, and firms compete in quantities in the input market,

$$C(t_o, t_r) = t_o Z(t_o + t_r).$$

It is immediate that, substituting  $-C(t_o, t_r)$  for  $R(t_o, t_r)$ , and  $-D(t)$  for  $S(t)$ , this model is homomorphic to our baseline model. Thus, the equilibrium conditions for the benchmark and market-clearing cases will be  $C_1(t, t) = D(2t)$  and  $C_1(t, t) -$

$C_2(t, t) = D(2t)$ , respectively. Of course, now  $C_2$  is positive – a firm’s costs are higher if its competitor buys more input – so  $C_1(t, t) - C_2(t, t) < C_1(t, t)$ . As the (inverse) demand is downward sloping, this will lead to higher overall production when firms engage in competitive foreclosure. Similarly to the input competition case, we can also sustain non-market-clearing equilibria, where there is unsatisfied demand (see Figure 3).

INSERT FIGURE 3 HERE

## 8.2 Competition for points of sale

Competition for exclusive retailers (or shelf space) is affected by similar vertical-linkage incentives but in reverse. As an illustration, consider again our linear Dixit’s (1979) model, but suppose that the number of “varieties” – or locations – is endogenous. Each “variety”  $i$  corresponds to the good sold by one retailer, and the representative consumer has a utility function  $U(m, \mathbf{q}) = m + \sum_i (aq_i - \frac{b}{2}q_i^2) - \frac{c}{2} \sum_i q_i \sum_{j \neq i} q_j$ , where  $\mathbf{q}$  is the vector of all quantities. The resulting inverse demand for retailer  $i$ ’s product/service is still linear,  $p^i(\mathbf{q}) = a - bq_i - cQ_{-i}$ , where  $Q_{-i}$  is the total output of all varieties other than  $i$ . Retailers – who secure supply – compete in quantities. They buy their necessary input/product from one of our two firms – manufacturers – who produce a homogeneous good at zero production cost. Firm  $i$  incurs a cost of dealing with  $t_j$  retailers,  $C(t_j)$ . Firms compete for retailers by asking for a personalized (two-part tariff with a zero variable part and fixed) payment,  $w_i$ . The manufacturer’s payoff is the difference between its tariff revenue and the cost  $C(\cdot)$ . (Thus, we are assuming homogeneous manufactured product/input and – symmetric – differentiated retailers.) Solving the simple quantity competition among retailers, we obtain retailer  $i$ ’s profit (gross of tariff payments) as

$$\pi(t) = \left( \frac{a}{2b + (t-1)c} \right)^2 b.$$

It is now straightforward to repeat our analysis to find that, if each firm took the rival’s number of retailers as given, then the “competitive” equilibrium would be

characterized by  $\pi'(2t)t + \pi(2t) - C'(t) = 0$  – and each firm would ask its retailers for a tariff  $w = \pi(2t)$ . As in our baseline model, this would not be an equilibrium in the game with personalized offers when manufacturers take into account externalities. Indeed, in that case manufacturer 1 would be willing to make such offer to one retailer that receives no other offer, but would be willing to undercut manufacturer 2 as long as  $\pi(t_1 + t_2) > C'(t_1)$ . The market clearing equilibrium would then be characterized by  $\pi(2t) - C'(t) = 0$  – and each firm would ask the same  $2t$  retailers for a tariff  $w = \pi(2t)$ . Since  $\pi' < 0$ , this means that, in equilibrium, more retailers will be enlisted – and more output produced.<sup>35</sup>

## 9 Conclusion

We have explored the consequences of linkages between market power in the output market and outcomes in upstream markets. When suppliers are viewed as separate markets, firms have incentives to attempt to restrict rivals' access to input, even when it is in abundant supply. The resulting competitive foreclosure leads to higher input prices and higher input levels, alleviating the anti-competitive effect of (downstream) market power.

There are a number of ways in which our model could be generalized. For example, note that we need not fully specify which subset of suppliers the firms choose to bid for. Obviously, a number of productivity-irrelevant characteristics (gender, race, first letter of last name, etc.) could serve this purpose and in that respect our model could also be a useful vehicle for modelling discrimination (c.f. Mailath et al., 2000).

An extension of the analysis to more than two firms could add further interesting consequences, for example segmentation of the input markets with different prices being paid for identical input, depending on the subset of competitors for them.

Finally, interpreting  $p$  as the per unit price offered, our model accommodates

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<sup>35</sup>Sufficient convexity of  $C$  or sufficiently large  $a$  relative to  $b$  and  $c$ , (so that  $3tc > b$ ) guarantee that this value is well defined.

(small) suppliers with heterogeneous capacities (or workers of different productivities). Similarly, we could also consider a finite number of suppliers. Such a reinterpretation comes with a cost though: Not only does the math become messy because of indivisibilities, but we can no longer assume that the firms know *ex ante* the quantity they will purchase in equilibrium. This uncertainty could become relevant in a dynamic extension of our model, since *ex ante* uncertainty about the equilibrium input quantity would give rise to a desire for further trade *ex post*, unlike in our base model.

It is revealing to note that the targetability of offers is not necessary for the high price/input-level result. Rather, it is the explicit consideration of each supplier's decision over which offer to accept that matters. Targetability does lead to additional equilibria, which exhibit even higher prices, but lower input level, leading to excess supply. Due to the foreclosure effect, firms are led to endogenously coordinate and make offers to the same subset of suppliers, while their willingness to pay for an uncontested supplier is strictly lower. From an applied point of view, our analysis underscores the importance of carefully understanding the interplay of market-power rents and upstream competition for the markets involved. Competition for inputs may not only transfer rents from downstream firms to suppliers, but it may actually reduce the size of these rents, and in so doing restore, at least partially, efficiency. It is well understood that assessing the effects and costs of market power, for instance in merger analysis, cannot be satisfactorily done without paying due attention to how the involved firms compete for their suppliers. This paper highlights one so far neglected intricacy that may characterize this competition.

# Appendix

Proof of Lemma 1:

**Proof.** Assume, by way of contradiction, that in equilibrium and in expectation a measure  $\alpha > 0$  of suppliers accept offers in  $[0, b]$  and a positive measure of suppliers accept offers in  $[c, d]$  for some  $0 \leq b < c \leq d$ . Take a firm that offers in  $[c, d]$  to a positive measure and denote by  $\beta > 0$  the measure of suppliers it enlists at these prices in expectation. Similarly, denote by  $\delta \in [0, \alpha]$  the expected measure of suppliers this same firm enlists for prices in  $[0, b]$ . Let  $\gamma = \min(\alpha - \delta, \beta)$ . Assume first that  $\delta < \alpha$ , and therefore the rival enlists some suppliers for prices in  $[0, b]$ . Also,  $\gamma > 0$ . Now if our firm deviates and outbids its rival in  $[0, b]$  – by  $\varepsilon < c - b$  – for enough suppliers so that it ends up in expectation with  $\delta + \gamma$  suppliers for prices in  $[0, b + \varepsilon]$ , while it withdraws enough offers from  $[c, d]$ , so that it enlists  $\beta - \gamma$  from that interval, it increases its expected payoff: net revenues from the product market either stay the same (if the suppliers given up in  $[c, d]$  go to the rival, both firms will enlist the same amount of suppliers as before) or increase (if the rival's amount of input is decreased due to the deviation) but the input bill is strictly lower. If  $\delta = \alpha$ , and therefore  $\gamma = 0$ , then the other firm does not enlist any supplier at wages in  $[0, b]$ , while, by the above argument, it cannot be making offers above  $b$  either, implying that it does not buy input at all, contradicting the assumption that both firms are active. ■

Proof of Proposition 1:

**Proof.** First, we show that firms offering  $p^c$  to  $2t^c$  suppliers in such a way that each supplier with reservation price below  $p^c$  receives exactly two offers – of which she chooses one with equal probabilities – is indeed an equilibrium. Suppose that Firm 2 behaves according to the hypothetical equilibrium strategy, and consider the best response of Firm 1:  $P_1(\cdot)$ . There are two types of supplier to target: there are measure  $2t^c$  suppliers with an offer of  $p^c$  and a reservation price less than that, and the rest of the suppliers who have a reservation price above  $p^c$ . Obviously, the firm should hire suppliers in increasing order of their – now perhaps determined by Firm 2's offer – reservation price, until this price equals marginal revenue. Thus, by the

definition of  $t^c$ , the equilibrium strategy is indeed a best response.

We now show that there exists no other equilibrium outcome (with deterministic price schedules). Assume, by way of contradiction, that there is a positive measure of suppliers who get a price strictly below  $p^c$  in equilibrium. Then there must exist a firm who would be willing to change a positive measure of its offers and instead offer  $\varepsilon$  more to these suppliers, as the aggregate demand at infra-competitive reservation prices strictly exceeds the supply of suppliers. Consequently (almost) no supplier can be enlisted for less than  $p^c$ . Similarly, assume, by way of contradiction, that there is a positive measure of suppliers who get a price strictly above  $p^c$ . Then there must be suppliers with a reservation price strictly below  $p^c$  who do not receive an acceptable offer, as the aggregate demand at supra-competitive reservation prices is strictly less than the supply of suppliers. Consequently, (almost) all the enlisted suppliers must be paid  $p^c$ . As no firm is willing to acquire more than its competitive demand at  $p^c$ , there are always suppliers willing to sell at this price, so each firm must buy up to its competitive quantity. ■

Proof of Proposition 2:

**Proof.** To see that  $(t^c, p^c)$  cannot be sustained by an equilibrium, note that as  $p^c = R_1(t^c, t^c) < R_1(t^c, t^c) - R_2(t^c, t^c)$ , any firm would prefer to outbid its rival by  $\varepsilon$  on some of the suppliers and lower its offer to zero for other, so that in expectation it buys the same amount of input, but the rival buys strictly less. The rest of the proposition follows from Proposition 3, proven below.<sup>36</sup> ■

Proof of Proposition 3:

**Proof.** Note that Assumption 1, ii) and iii) and  $S' \geq 0$ , ensure that  $(t^*, p^*)$  is well defined. We prove the first part of the proposition by contradiction. Suppose there is no interval  $[t, t^*]$  such that for each  $t$  in the interval the equilibrium depicted in the

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<sup>36</sup>Note that Assumption 1iii) guarantees that profits are non-negative at  $t^*$ . Indeed,

$$R(t^*, t^*) - w^* t^* = \int_0^{t^*} (\Phi(x) - R_1(t^*, t^*) - R_2(t^*, t^*)) dx$$

where  $\Phi(x) = R_1(x, 2t^* - x) - R_2(x, 2t^* - x)$ , and the result follows noticing that  $\Phi(x)$  is decreasing from Assumption 1iii) and the inside of the integral is zero at  $x = t^*$ .

proposition exists. That implies that there exists a sequence  $t_n \uparrow t^*$ , where, for all  $n$ , it is not an equilibrium that both firms offer a price  $p_n = R_1(t_n, t_n) - R_2(t_n, t_n)$  to the  $2t_n$  suppliers with lowest reservation price, who then accept each of them with probability 0.5. Clearly, no supplier has a profitable deviation in this case. Thus, it must be that one firm, say Firm 1, does. The response of Firm 1 amounts to finding quantities  $\alpha$  of input acquired from the pool of  $2t_n$  who receive offers from Firm 2, and  $\beta$  of input acquired from the pool that does not receive offers from Firm 2. Indeed, that can always be done by offering  $p_n$  to  $2\alpha$  of the former<sup>37</sup> and their reservation price to suppliers in the interval  $(2t_n, 2t_n + \beta)$ . For given  $\alpha$  and  $\beta$ , the profit of Firm 1 is then (arbitrarily close to)

$$\pi^1(\alpha, \beta) = R(\alpha + \beta, 2t_n - \alpha) - p_n \alpha - \int_{2t_n}^{2t_n + \beta} S(x) dx, \quad (4)$$

with derivatives

$$\frac{d\pi^1(\alpha, \beta)}{d\alpha} = R_1(\alpha + \beta, 2t_n - \alpha) - R_2(\alpha + \beta, 2t_n - \alpha) - p_n \quad (5)$$

and

$$\frac{d\pi^1(\alpha, \beta)}{d\beta} = R_1(\alpha + \beta, 2t_n - \alpha) - S(2t_n + \beta). \quad (6)$$

Note that  $\frac{d\pi^1(\alpha, 0)}{d\alpha} = R_1(\alpha, 2t_n - \alpha) - R_2(\alpha, 2t_n - \alpha) - p_n = R_1(\alpha, 2t_n - \alpha) - R_2(\alpha, 2t_n - \alpha) - R_1(t_n, t_n) + R_2(t_n, t_n)$ . Since  $\frac{d\pi^1(t_n, 0)}{d\alpha} = 0$ , and  $R$  is concave in  $\alpha$  from Assumption 1iii), we conclude that a best response given  $\beta = 0$  is  $\alpha = t_n$ . Thus, if there is a better response than the putative equilibrium strategy,  $\alpha = t_n$ ,  $\beta = 0$ , it must be with  $\beta > 0$ .

Let the sequence of best responses to  $\{t_n\}$  be denoted  $\{\alpha_n, \beta_n\}$ . This sequence is bounded, since  $R(t^*, t^*) - pt^* > 0$  and  $\pi^1(\alpha, \beta) < 0$  for large enough  $\alpha$  and/or  $\beta$ . Thus, it has accumulation points. Suppose  $(0, 0)$  is an accumulation point for this sequence. Then, for some  $n$  large enough  $\pi^1(\alpha_n, \beta_n)$  is arbitrarily close to  $R(0, 2t^*) = 0$ . This cannot be a best response, since  $\pi^1(t_n, 0)$  is arbitrarily close to  $R(t^*, t^*) - pt^* > 0$ .

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<sup>37</sup>If  $\alpha > t_n$  then Firm 1 can offer  $p + \varepsilon$  to  $\alpha$  of these suppliers.

Thus, at any accumulation point  $(\hat{\alpha}, \hat{\beta})$ , we have  $\hat{t} = \hat{\alpha} + \hat{\beta} > 0$ . That means that there exists a subsequence of  $\{\alpha_n, \beta_n\}$  such that for  $n$  large  $R_2(\alpha_n + \beta_n, 2t_n - \alpha_n)$  is arbitrarily close to  $R_2(\hat{t}, 2t^* - \hat{\alpha}) < 0$ . Since  $\beta_n > 0$ , (6) equals zero evaluated at  $(\alpha_n, \beta_n)$ , so that  $R_1(\hat{t}, 2t^* - \hat{\alpha}) = S(2t^* + \hat{\beta}) \geq S(2t^*)$ . Also, since (5) is not larger than zero at these values (zero if  $\alpha_n > 0$ ),  $R_1(\hat{t}, 2t^* - \hat{\alpha}) - R_2(\hat{t}, 2t^* - \hat{\alpha}) \leq \bar{p} = R_1(t^*, t^*) - R_2(t^*, t^*) = S(2t^*)$ . As  $R_2(\hat{t}, 2t^* - \hat{\alpha}) < 0$ , the two previous inequalities are contradictory. Therefore, there cannot exist a sequence  $t_n \uparrow t^*$ , where, for all  $n$ , Firm 1 has a better response than the proposed strategy when Firm 2 follows that strategy. This in turn proves that there has to exist a non-degenerate interval  $[\underline{t}, t^*]$  in which the proposed strategies are indeed an equilibrium.

We now show that there is no other class of symmetric equilibria in pure price schedules. By Lemma 1 the offers by both firms are identical for all the suppliers that are enlisted. Suppose all these suppliers receive an offer different from  $R_1(t, t) - R_2(t, t)$ , where  $t$  is the amount of input acquired by each firm. If the price offered is larger than  $R_1(t, t) - R_2(t, t)$ , then a firm would profit from withdrawing some of its offers, whereas if it is lower than that amount, then a firm would profit from increasing its offers to a small mass of suppliers that accept with positive probability its rival's offers, and so "stealing" them from its rival. Thus, both firms must offer the same price  $p = R_1(t, t) - R_2(t, t)$  to  $2t$  suppliers, and each must hire half of them. These  $2t$  suppliers must accept, so their reservation price must be below  $p$ . This completes the proof. ■

Proof of Proposition 4:

**Proof.** Let Firm 2 play its equilibrium strategy, and so offer  $p$  to a number of suppliers  $q_2$ .<sup>38</sup> Consider Firm 1's strategy consisting of sending offers with the same price  $p$  (say,  $b$  of them) or – slightly – higher ones (say,  $a$  of them), and no offers with lower prices. Any of the  $a$  offers would be accepted by the supplier receiving it if her reservation price is lower than  $p$ , whereas each of the  $b$  ones would be accepted

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<sup>38</sup>For a symmetric equilibrium with prices  $p$ , any offer below or above  $p$  must be rejected. Offers above – since offers cannot be targeted – will always have a positive probability of being accepted. Offers below too, unless the rival firm offers price  $p$  to *all* workers. Symmetry would imply that both firms do that, eliminating the possibility of offers below  $p$  too.



with probability  $\frac{2T-q_2}{2T}$  by these same suppliers. Indeed, since the equilibrium is symmetric, half the suppliers who receive both offers must choose Firm 1's offer in equilibrium. Moreover, and more importantly, since suppliers cannot be targeted by assumption, even after the deviation by Firm 1, half of those who still receive the same offer from both firms must choose Firm 1's. The probability that any given offer of  $p$  made by Firm 1 is to a supplier who also receives an offer from Firm 2 is  $\frac{q_2}{T}$ . In addition, the probability that the offer is to a supplier who does not receive an offer from Firm 2 is  $\frac{T-q_2}{T}$ , and then the offer will be accepted if  $p$  exceeds the supplier's reservation price. Similarly, we can compute the probability that Firm 2's offer is accepted by a supplier with reservation price below  $p$  when these strategies are used, as  $\frac{2(T-a)-b}{2T}$ . Thus, for  $a = 0$ ,  $b > 0$  to be a best response for Firm 1, these values would have to maximize<sup>39</sup>

$$R\left(\frac{S^{-1}(p)}{T}\left[a + \frac{2T-q_2}{2T}b\right], \frac{S^{-1}(p)}{T}\left[\frac{2(T-a)-b}{2T}q_2\right]\right) - p\frac{S^{-1}(p)}{T}\left(a + \frac{2T-q_2}{2T}b\right).$$

The first order conditions for  $a = 0$ ,  $b > 0$  require

$$R_1 - \frac{q_2}{2T-q_2}R_2 = p \geq R_1 - \frac{q_2}{T}R_2,$$

where we have omitted the arguments of the functions. Since  $R_2 < 0$ , these conditions are incompatible unless  $q_2 = T$ . For  $b = q_2 = T$ , the input acquired by each firm is  $t = \frac{S^{-1}(p)}{2}$ , so that we have demand equal to supply and the first order conditions imply

$$R_1(t, t) - R_2(t, t) = p = S(2t),$$

as we wanted to show. ■

Proof of Proposition 5:

**Proof.** We will show that there exist  $t$  and  $p$  with  $S^{-1}(p) > 2t$ , and a symmetric equilibrium where both firms contract input  $t$  in the first period with offers  $p$ , and

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<sup>39</sup>The expression is written as if all offers  $a$  included price offer  $p$ . Note that the profits obtained by the firm with  $a$  offers just above  $p$  can be arbitrarily close to that expression.

send no new offers in the second period. For that to be the case, each firm must send  $q$  offers of  $p$ , satisfying

$$t = q \frac{S^{-1}(p)}{T} \cdot \frac{2T - q}{2T}. \quad (7)$$

Suppose that Firm 2 does so, but Firm 1 deviates and sends  $q^1$  offers with price  $p^1$ . Consider  $p^1 \geq p$ , first. It is straightforward that such a deviation can only be profitable if it leads Firm 1 to acquire  $t^1 = \frac{q^1}{T} S^{-1}(p^1) > t$ . Firm 1's deviation also affects how much input Firm 2 obtains:

$$t^2(t^1, p^1) = q \frac{S^{-1}(p)}{T} \left[ 1 - \frac{t^1}{S^{-1}(p^1)} \right]. \quad (8)$$

The first term in the right hand side is the measure of offers sent by Firm 2, the second is the probability that a supplier that receives one has a reservation price below  $p$ , and the third term is the probability that one such supplier has not received one of the offers sent by Firm 1.

First, it may be Firm 1 who has a chance to send new offers. Then it solves the problem

$$\max_{\Delta^1 \geq 0} \{ R(t^1 + \Delta^1, t^2(t^1, p^1) - \Delta^1) - p \Delta^1 \}.$$

The first order condition is

$$\Phi = R_1(t^1 + \Delta^1, t^2(t^1, p^1) - \Delta^1) - R_2(t^1 + \Delta^1, t^2(t^1, p^1) - \Delta^1) - p = 0. \quad (9)$$

The objective function is concave (in  $\Delta^1$ ) and continuous in  $[0, t^2(t^1, p^1)]$ . Let the solution be  $\hat{\Delta}^1$ . Observe that

$$\frac{d\hat{\Delta}^1}{dp^1} = - \frac{\frac{\partial t^2}{\partial p^1} [R_{12} - R_{22}]}{\partial \Phi / \partial \Delta^1} \leq 0,$$

since the denominator is negative and  $\frac{\partial t^2}{\partial p^1} > 0$ .

Now assume it is Firm 2 who can make new offers. It solves a similar problem, namely

$$\max_{\Delta^2 \in [0, t^1]} R(t^2(t^1, p^1) + \Delta^2, t^1 - \Delta^2) - p^1 \Delta^2.$$

This optimization problem is also concave in  $\Delta^2$ , with first order condition

$$R_1(t^2(t^1, p^1) + \Delta^2, t^1 - \Delta^2) - R_2(t^2(t^1, p^1) + \Delta^2, t^1 - \Delta^2) - p^1 = 0. \quad (10)$$

Let the solution be  $\tilde{\Delta}^2$ . Similarly as before,  $\frac{d\tilde{\Delta}^2}{dp^1} \leq 0$ .

Let us now consider the best deviation (with  $p^1 > p$ ) by Firm 1 in period 1. The optimal  $(p^1, t^1)$  solves

$$\max_{p^1 \geq p, t^1 \geq t} \frac{1}{2} \{ R(\hat{t}^1, \hat{t}^2) - p^1 t^1 - p(\hat{t}^1 - t^1) + R(\tilde{t}^1, \tilde{t}^2) - p^1 \tilde{t}^1 \},$$

where  $\hat{t}^1 = t^1 + \hat{\Delta}^1$ ,  $\hat{t}^2 = t^2(t^1, p^1) - \hat{\Delta}^1$ ,  $\tilde{t}^1 = t^1 - \tilde{\Delta}^2$ , and  $\tilde{t}^2 = t^2(t^1, p^1) + \tilde{\Delta}^2$ . The derivative of the objective function with respect to  $p^1$  is

$$\begin{aligned} & \frac{1}{2} \left[ -t^1 + \frac{\partial \hat{t}^1}{\partial p^1} (R_1(\hat{t}^1, \hat{t}^2) - p) + \frac{\partial \hat{t}^2}{\partial p^1} R_2(\hat{t}^1, \hat{t}^2) \right] \\ & + \frac{1}{2} \left[ -\tilde{t}^1 + \frac{\partial \tilde{t}^1}{\partial p^1} (R_1(\tilde{t}^1, \tilde{t}^2) - p^1) + \frac{\partial \tilde{t}^2}{\partial p^1} R_2(\tilde{t}^1, \tilde{t}^2) \right]. \end{aligned}$$

Note that  $\frac{\partial \hat{t}^1}{\partial p^1} = \frac{d\hat{\Delta}^1}{dp^1} = \frac{\partial t^2}{\partial p^1} - \frac{\partial \hat{t}^2}{\partial p^1}$ . Also, note that  $\frac{\partial \tilde{t}^2}{\partial p^1} = \frac{d\tilde{\Delta}^2}{dp^1} + \frac{\partial t^2}{\partial p^1} = -\frac{\partial \tilde{t}^1}{\partial p^1} + \frac{\partial t^2}{\partial p^1}$ .

Thus, the expression above can be written as

$$\begin{aligned} & \frac{1}{2} \left[ -t^1 + \frac{d\hat{\Delta}^1}{dp^1} (R_1(\hat{t}^1, \hat{t}^2) - R_2(\hat{t}^1, \hat{t}^2) - p) + \frac{\partial t^2}{\partial p^1} R_2(\hat{t}^1, \hat{t}^2) \right] \\ & + \frac{1}{2} \left[ -\tilde{t}^1 - \frac{d\tilde{\Delta}^2}{dp^1} (R_1(\tilde{t}^1, \tilde{t}^2) - R_2(\tilde{t}^1, \tilde{t}^2) - p^1) + \frac{\partial t^2}{\partial p^1} R_2(\tilde{t}^1, \tilde{t}^2) \right]. \end{aligned}$$

As we mentioned,  $\frac{d\tilde{\Delta}^2}{dp^1} \leq 0$ , and  $\frac{\partial t^2}{\partial p^1} > 0$  in (8). Also, note that  $R_1(\hat{t}^1, \hat{t}^2) - R_2(\hat{t}^1, \hat{t}^2) = p$ , from (9). Thus, this whole expression can be seen to be negative if

$$R_1(\tilde{t}^1, \tilde{t}^2) - R_2(\tilde{t}^1, \tilde{t}^2) - p^1 \leq 0.$$

Note that  $\tilde{t}^1 + \tilde{t}^2 = \hat{t}^1 + \hat{t}^2 = t^1 + t^2(t^1, p^1)$ , and so if  $\tilde{t}^1 > \tilde{t}^2$ , just as in the case of (9), the inequality follows. Thus, we must have an optimizer at  $p (+\epsilon)$  if  $\tilde{t}^1 \geq \tilde{t}^2$ . Note that, once again resorting to Assumption 1, taking into account the first – and second– order condition for the optimal choice of Firm 2 (10) and the fact that  $\tilde{t}^1 + \tilde{t}^2 = t^1 + t^2(t^1, p^1) \geq 2t$ , indeed  $\tilde{t}^1 \geq \tilde{t}^2$ .

Thus, any (optimal) deviation simply sets an amount of input contracted,  $t^1 + t^2(t^1) \geq 2t$ , and then results in each firm choosing how much of it to acquired at wage  $p$  with probability 1/2. I.e., the best deviation results in a symmetric (expected)

situation, and so in the same expected profits, for both firms for each  $t^1$ . In the region with  $t^1 \geq t$ , and so  $t^1 + t^2(t^1) \geq 2t$ , the expected profits are largest when,  $t^1 = t$ , and so  $t^2(t^1) = t$ , since for any  $t' > t$ ,

$$\begin{aligned} R(t, t) - pt &> R(t', t') - pt' \geq \\ &\geq \frac{1}{2}R(t' + \delta, t - \delta) + \frac{1}{2}R(t' - \delta, t + \delta) - t'p. \end{aligned}$$

Indeed, the first inequality follows from the fact that at given price  $p$ , the derivative of  $R(t', t') - pt'$  with respect to  $t'$  is

$$R_1(t', t') + R_2(t', t') - p < R_1(t', t') - R_2(t', t') - p,$$

and  $R_1(t', t') - R_2(t', t')$  is decreasing in  $t'$ . Thus, for  $p = R_1(t, t) - R_2(t, t)$ , that derivative is negative. The second follows from Assumption 1iii), i.e., concavity of  $R(t' + \alpha, t' - \alpha)$  in  $\alpha$ .

Now, consider a deviation with  $p^1 < p$ . Given  $p^1$ ,  $t^1$  would be given by

$$t^1 = S^{-1}(p^1) \frac{q^1}{T} (1 - \frac{q}{T}).$$

That is,  $t^1$  can be chosen to be any number in  $\left[0, \frac{S^{-1}(p^1)}{T}(T - q)\right]$ . Observe that (7) implies that for  $p$  sufficiently close to  $p^*$ ,  $q$  is arbitrarily close to  $T$ , so that this interval is arbitrarily small. That is,  $t^1$  is arbitrarily small and so  $R(t^1, t^2) - p^1 t^1$  is arbitrarily small. That is, smaller than  $R(t, t) - pt$ . Also, the maximum of  $R(t^1 + \Delta, t^2 - \Delta) - p^1 t^1 - p\Delta$  in  $\Delta$  is attained arbitrarily close to  $\Delta = t$ , since  $t^1 + t^2$  is arbitrarily close to  $2t$ . Thus, the losses in case it is Firm 2 who is allowed to send new offers more than offset any possible gains in case Firm 1 is allowed to readjust input. This proves the result. ■

Proof of Corollary 2

**Proof.** Suppose Firm 2 sends  $2t$  offers with price  $p = a - 2bt$ , and let  $x$  represent the total input that Firm 1 obtains, and  $\alpha$  the input that it obtains (by sending offers  $\sim p$  but larger than  $p$ ) from the pool of suppliers that receive offers from Firm 2. Since the uncontested suppliers are paid  $r$ , Firm 1's profit  $\pi^1(x, \alpha; t)$  has a positive slope in  $\alpha$  if and only if (c.f. (4) in the proof of Proposition 3)

$$x > \frac{p - r}{c} \iff t > t^c = \frac{a - r}{2b + c}.$$

Thus, given  $x$ , the optimal  $\alpha$ ,  $\alpha(x)$ , equals  $x$  if the inequality is satisfied and 0 if it is reversed.<sup>40</sup> Also,  $\pi^1(x, \alpha(x); t)$ , is concave in  $x$  both to the right and to the left of  $\frac{p-r}{c}$ , and its derivative to the right of that value – when  $\alpha(x) = x$  – is zero at  $x = t$ . Thus, we have a symmetric equilibrium if and only if the left derivative with respect to  $x$ , evaluated at  $x = \frac{p-r}{c}$  is non-negative. This is the case if

$$t \geq \underline{t} = \frac{a - r}{2b + c - \frac{c^2}{2b-c}}.$$

■

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<sup>40</sup>It is interesting to observe that the binding deviation from a symmetric putative equilibrium with a little lower measure of labor hired is not to hire more workers (either contested or uncontested) rather, to drop all offers to contested workers and hire some – but fewer – uncontested ones.

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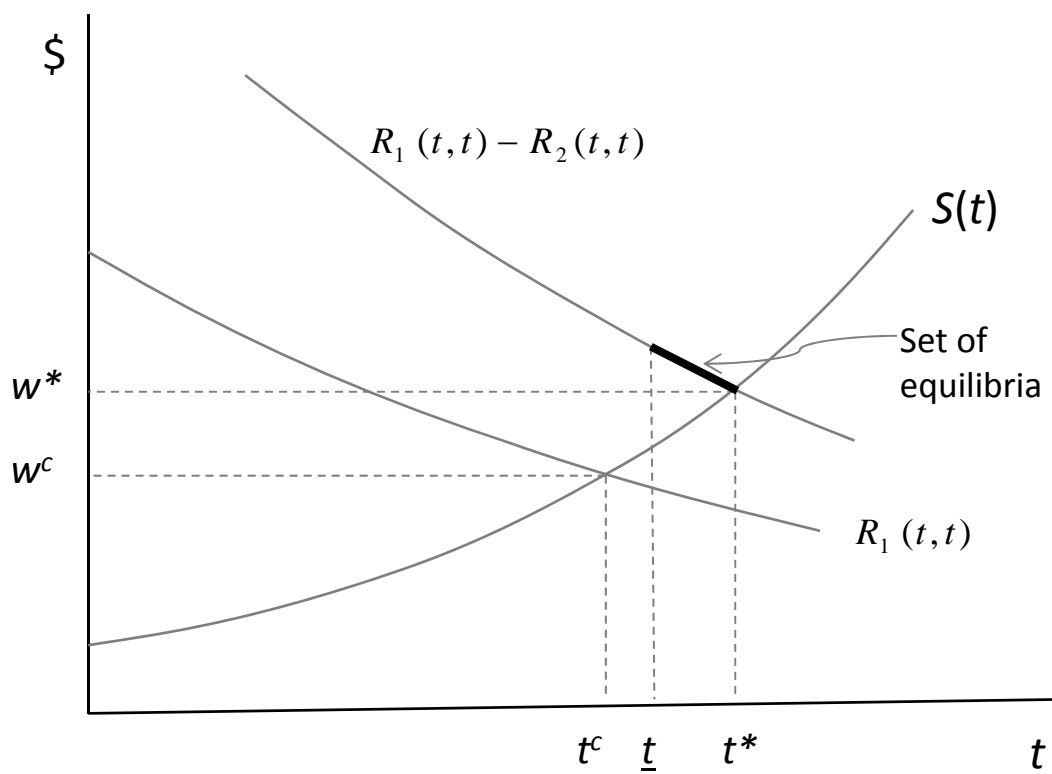


Figure 1

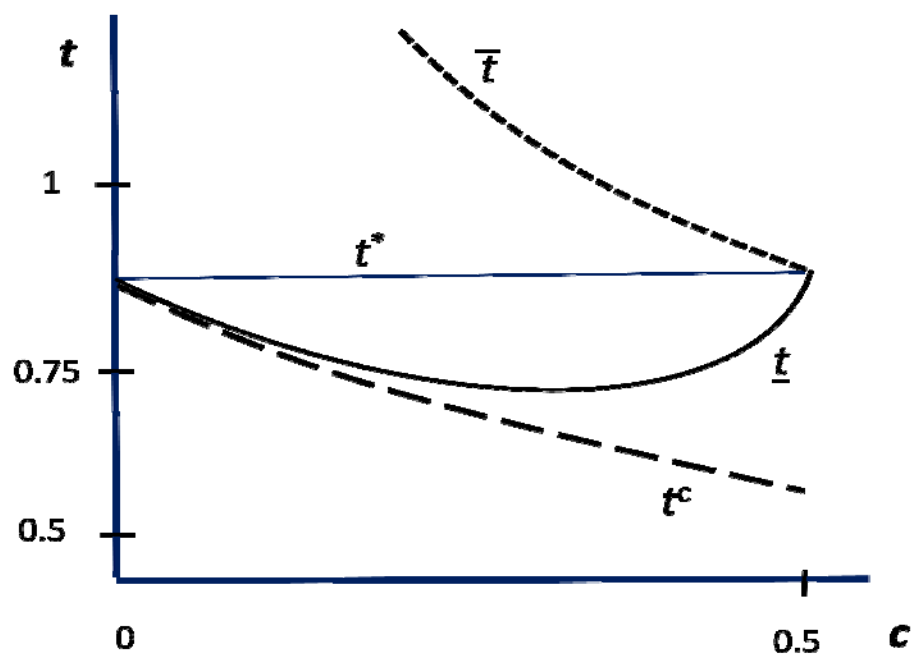


Figure 2

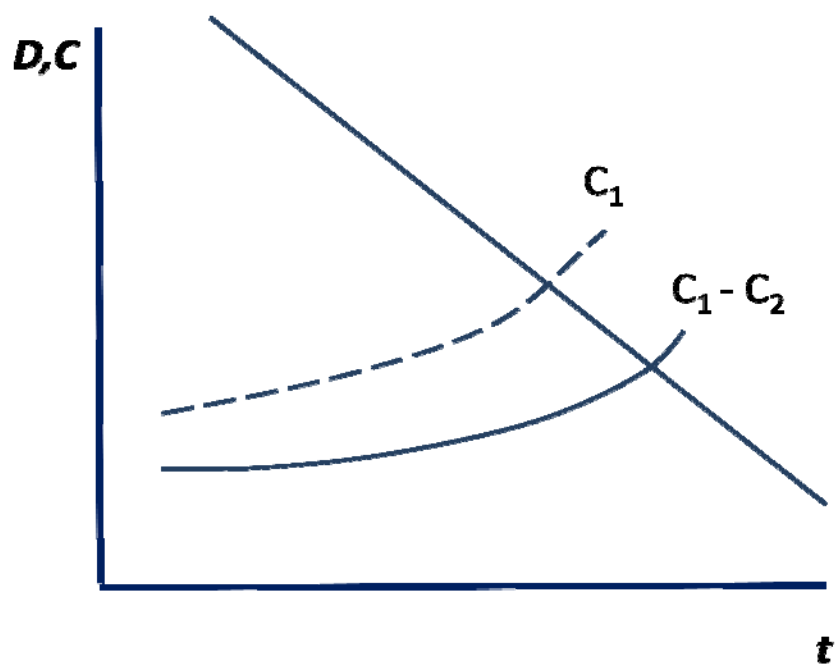


Figure 3